

B.Sc. Semester - 1 (CBCS) Examination
Oct/Nov-2023 (Syllabus Year - 2016)
MATHEMATICS (CORE)

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

- Que.1(A) Answer any one out of two. (07)
1. In usual notation prove that $e^x \cdot \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{5} - \dots$
 2. Expand $e^x \sin^2 x$ up to first three terms in ascending power of x .
- Que.1(B) Answer any one out of two. (04)
1. For $x > 0$, show that $\frac{x}{1+x^2} < \tan^{-1} x < x$.
 2. Find coefficient of x^4 in expansion of $\log \cos x$.
- Que.1(C) Answer any three out of five. (03)
1. Write the formula of Lagrange's theorem.
 2. Find the value of c , if Roll's theorem is applicable to $f(x) = x^2 - 3x + 2$, $x \in [1, 2]$.
 3. Check whether the function $f(x) = 6(x-2)^2$, $x > 2$ is increasing or decreasing.
 4. Write expansion of e^x in ascending power of x .
 5. Show that $f(x) = x^3 + 6x^2 + 12x - 7$ is strictly increasing in every interval.
- Que.2(A) Answer any one out of two. (07)
1. Solve: $\sqrt{1-y^2} dx = (\sin^{-1} y - x) dy$.
 2. Solve: $\frac{dy}{dx} = x^3 - 2xy$.
- Que.2(B) Answer any one out of two. (04)
1. $\lim_{x \rightarrow \pi/2} (\sin x)^{\tan x}$.
 2. $\lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{a - b \operatorname{cosec} x} = 1$ then find value of a and b .
- Que.2(C) Answer any three out of five. (03)
1. Evaluate $\lim_{x \rightarrow 0} \frac{x}{\tan 2x}$.
 2. Evaluate $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 3x}$.
 3. State general form of linear differential equation.
 4. Find general solution of $\sqrt{1+x^2} dy = \sqrt{1+y^2} dx$.
 5. Evaluate $\lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1}$.
- Que.3(A) Answer any one out of two. (07)
1. Solve $y = 2px + y^2 p^3$.
 2. Show that $(x^2 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y)dy = 0$ is exact, hence find its solution.
- Que.3(B) Answer any one out of two. (04)
1. Solve $p^2 - (x+3y)p + 2y(x+y) = 0$.
 2. Give general form of Clairaut's differential equation, also find its solution.
- Que.3(C) Answer any three out of five. (03)
1. State general form of Bernoulli's differential equation.
 2. Find the integrating factor of $x \frac{dy}{dx} = x + y$.
 3. Determine whether the differential eq. $ydx + (y+x)dy = 0$ is exact or not.
 4. Solve: $y = px + \sin^{-1} p$.
 5. Find the integrating factor of $x \frac{dy}{dx} + y = x^3 y^3$.
- Que.4(A) Answer any one out of two. (07)
1. Solve $(D^3 - 12D^2 + 6D - 8)y = e^{2x}$.
 2. In usual notation prove that $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$ if $f(a) \neq 0$.

- Que.4(B) Answer any one out of two. (04)
1. Solve $(D^2 - 7D + 12)y = e^{5x}$.
 2. Solve $(D^2 - 1)y = \sin 5x$
- Que.4(C) Answer any three out of five. (03)
1. Solve $(D^4 + 2D^2 + 1)y = 0$.
 2. Solve $(6D^2 - D - 2)y = 0$.
 3. Find $\frac{1}{D^2 - 7D + 12} e^x$
 4. Find $\frac{1}{(D-2)^3} e^{2x}$
 5. Find only P.I. for $(D^2 - 1)y = \sin 5x$
- Que.5(A) Answer any one out of two. (07)
1. Solve: $(x^2 D^2 + 7xD + 5)y = x^5$
 2. Solve: $(x^2 D^2 - 2xD + 2)y = x^3$
- Que.5(B) Answer any one out of two. (04)
1. Find only P.I. for $(x^2 D^2 + 2xD - 20)y = (x+1)^2$.
 2. Solve: $(xD^3 - D^2)y = 1/x$.
- Que.5(C) Answer any three out of five. (03)
1. Solve $(x^2 D^2 - xD - 3)y = 0$
 2. Solve $(x^2 D^2 + xD - 1)y = 0$
 3. Find C.F. of $(x^2 D^2 + 3xD + 1)y = x^2 e^x$
 4. Solve $(x^2 D^2 - 2xD + 2)y = 0$
 5. Find only P.I. for $(x^2 D^2 + 7xD + 5)y = x^3$
